CS 4649/7649
Robot Intelligence: Planning

Linear Control

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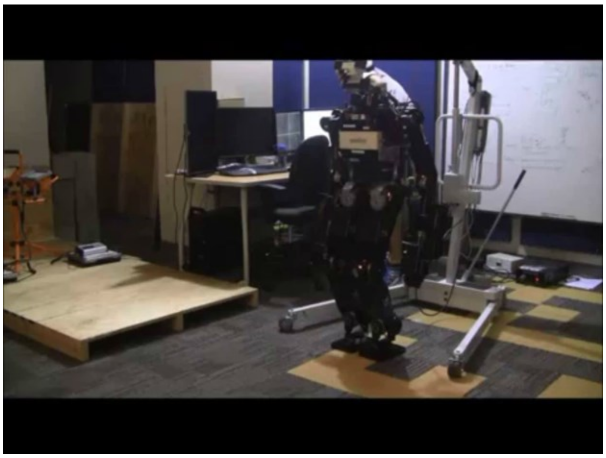
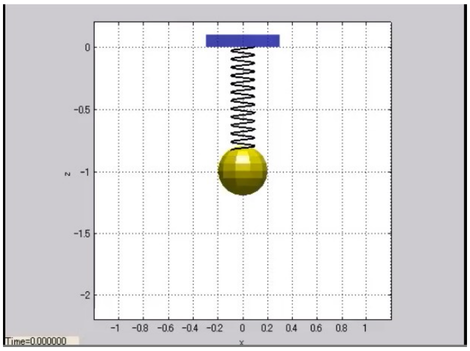
The Robot is a Dynamic System



The Robot is a Dynamic System

What is a Dynamic System?

- Mechanical systems, (analog) electrical circuit, …



The Robot is a Dynamic System

What is a Dynamic System?

- A system whose variables (input, state, output) are time-dependent

u(t) ⟶ [**x(t)**] ⟶ **y(t)**

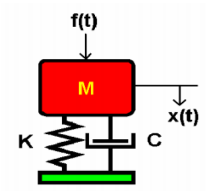
How can we describe a dynamic system?

The Robot is a Dynamic System

What is a Dynamic System?

- A system whose variables are time-dependent
- Linear system $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}, \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}$

How can we describe a dynamic system?



The Robot is a Dynamic System

What is a Dynamic System?

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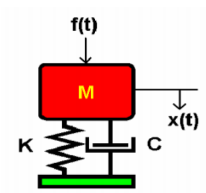
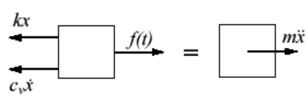
How can we describe a dynamic system?



Differential equations

$$\mathbf{M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t)}$$

Free-body diagrams



$$\mathbf{[M\,s^2 + C\,s + K]\;X(s) = F(s)}$$

Transfer functions

$$\mathbf{H(s) = \frac{X(s)}{F(s)} = \left(\frac{1}{M s^2 + C s + K}\right)}$$

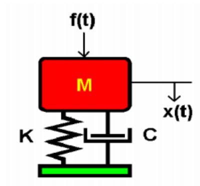
Block diagrams

f(t) → **Plant** → x(t)

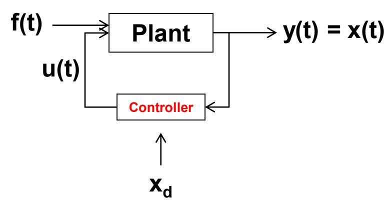
Control

What is Control?

Given the system state, choose input to get a desired output.



If we want to keep $x = x_d$, what should we do?



Car Speed Control

Cruise Control (Open Loop)

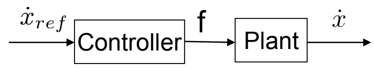
$$f - m\ddot{x} - b\dot{x} = 0$$

Steady State → Let $\ddot{x}_{ss} = 0$

$$0 + b\dot{x}_{ss} = f$$

$$\dot{x}_{ref} = \dot{x}_{ss} = \frac{f}{b}$$

$$f = b\dot{x}_{ref}$$

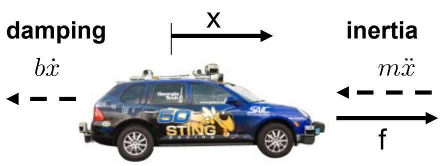


State Variable Form:

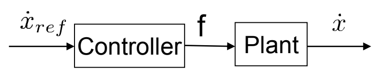
$$\ddot{x} = \left[-\frac{b}{m}\right]\dot{x} + \left[\frac{1}{m}\right]f$$

$$\ddot{x} = \left[-\frac{b}{m}\right]\dot{x} + \left[\frac{b}{m}\right]\dot{x}_{ref}$$

Cruise Control (Open Loop)



$$f - m\ddot{x} - b\dot{x} = 0$$



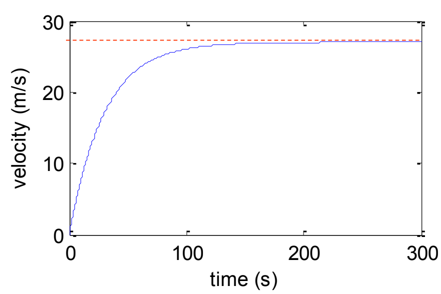
State Variable Form:

$$\ddot{x} = \left[-\frac{b}{m}\right]\dot{x} + \left[\frac{1}{m}\right]f$$

Advantages ?

Disadvantages ?

m = 1500kg b = 50Ns/m v_r = 27m/s



Car Steering Control

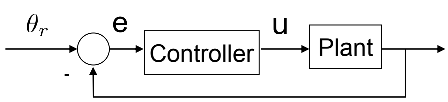
Steering Control (Proportional Control)

$$J\ddot{\theta} + b\dot{\theta} = \tau$$

$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau$$

$$\tau = -K_p(\theta - \theta_r)$$

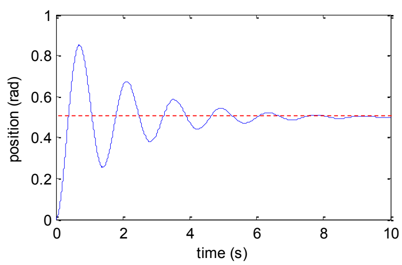
$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - K_p\frac{1}{J}(\theta - \theta_r)$$



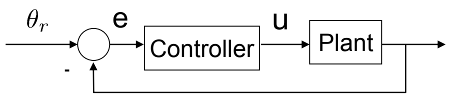
State Variable Form:

$$\begin{bmatrix}\dot{\theta} \\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -\frac{K_p}{J} & -\frac{b}{J}\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0 \\ \frac{K_p}{J}\end{bmatrix}\theta_r$$

J = .1kgm^2 b = .1Nms θ_r = .5rad



Car Steering (Proportional Control)



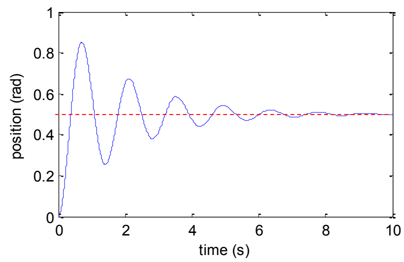
$$\tau = -K_p(\theta - \theta_r)$$

$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - K_p\frac{1}{J}(\theta - \theta_r)$$

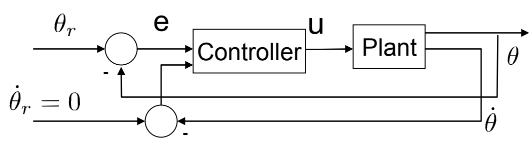
Stability?

(Exponential) Convergence?

Limitations?



Steering Control (PD Control)



State Variable Form:

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K_p}{J} & -\frac{b+K_d}{J}\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\ \frac{K_p}{J}\end{bmatrix}\theta_r$$

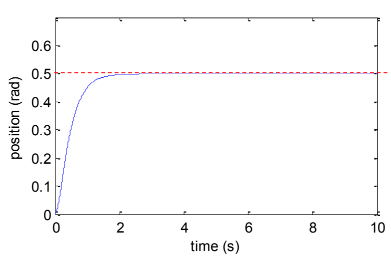
J = .1kgm^2 b = .1Nms θ_r = .5rad

$$J\ddot{\theta} + b\dot{\theta} = \tau$$

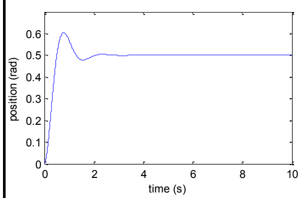
$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau$$

$$\tau = -K_d(\dot{\theta} - \dot{\theta}_r) - K_p(\theta - \theta_r)$$

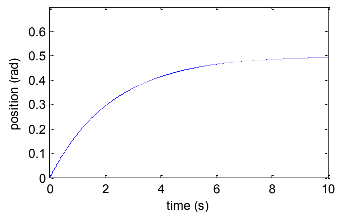
$$\ddot{\theta} = -\frac{b}{J}\dot{\theta} - K_d\frac{1}{J}(\dot{\theta} - \dot{\theta}_r) - K_p\frac{1}{J}(\theta - \theta_r)$$



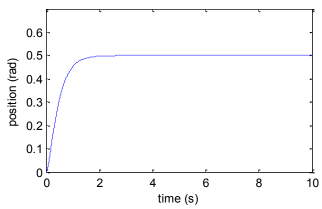
PD Control



Under-damped



Over-damped



Critically damped (Just Right)

PD Control (Selecting Gains)

$$\ddot{\theta} = -\frac{b + K_d}{J}\dot{\theta} - \frac{K_p}{J}(\theta - \theta_r)$$

$$\ddot{\theta} = -K_d^{'}\dot{\theta} - K_p^{'}(\theta - \theta_r)$$

→

$$\ddot{\theta} + K_d^{'}\dot{\theta} + K_p^{'}\theta = K_P^{'}\theta_r$$

Characteristic Polynomial: $s^2 + K_d^{'}s + K_p^{'}$

Standard Form: $s^2 + 2\zeta w_n s + w_n^2$

Damping Ratio: ζ
Natural Frequency: w_n

$$K_p^{'} = w_n^2$$

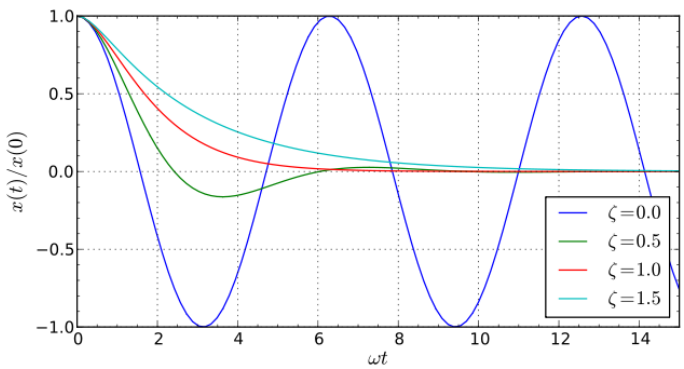
$$K_d^{'} = 2\zeta w_n$$

$\zeta = 1$ →

$$K_p^{'} = w_n^2$$

$$K_d^{'} = 2w_n = 2\sqrt{K_p^{'}}$$

Dynamic Response



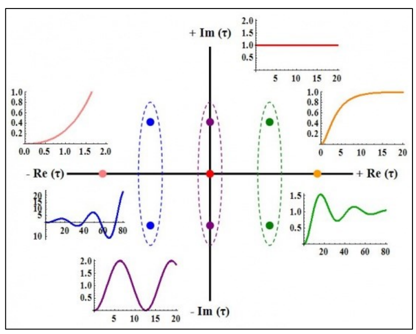
PD Control (Selecting Gains)

Starting Point: $K_p^{'} = w_n^2$

$$K_d^{'} = 2w_n = 2\sqrt{K_p^{'}}$$

What is w_n ?

Gains are limited by:
- structural flexibility
- time delay
- sampling rate
- actuator saturation



Response of 2nd order system (step input)

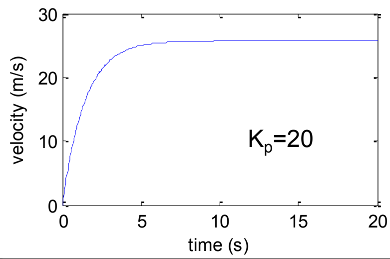
Optimal Control – Linear Quadratic Regulator

Guess/Nominal values → Tuning!

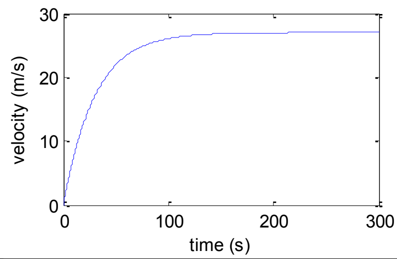
Cruise Control Revisited (Closed Loop)

$$f - m\ddot{x} - b\dot{x} = 0$$

$$f = -K_p(\dot{x} - \dot{x}_r)$$



Open Loop:



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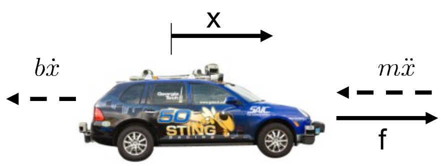
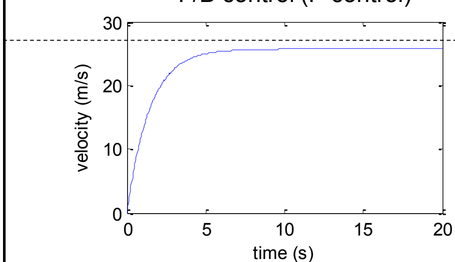
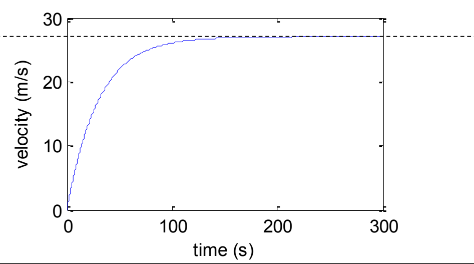


$$f - m\ddot{x} - b\dot{x} = 0$$

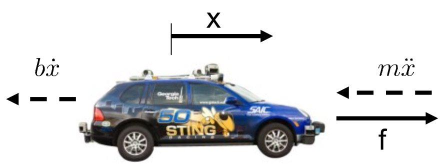
F/B control (P control)



Open Loop



Fix Steady State Error!



$$f - m\ddot{x} - b\dot{x} = 0$$

Option 1: Solve for the correct offset (use model)

Option 2: Integral Control (P → I ← D) … be careful!

Integral Action

PI controller

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)\, d\tau$$

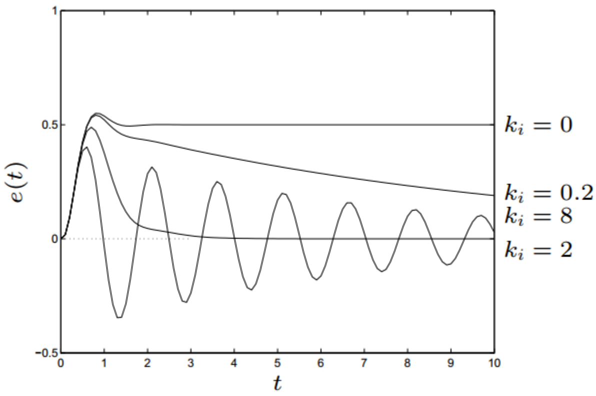
Add another 'corrective' or 'restoring' term, proportional to the integral of error

A constant error e
- yields a constant corrective reaction for proportional action
- yields a growing corrective reaction for integral action

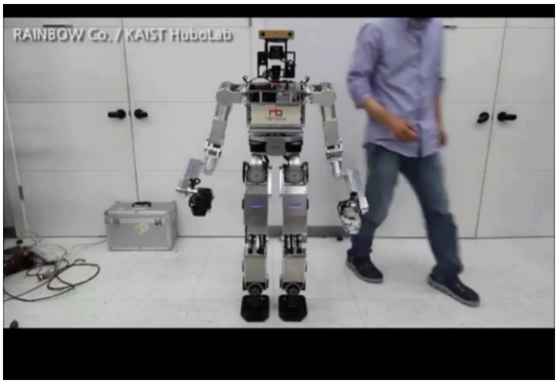
Choice of integral gain
- too small: get asymptotic tracking, disturbance rejection, but very slow
- too large: oscillatory response, or even instability

Integral Action

PI controller



Manipulator Control



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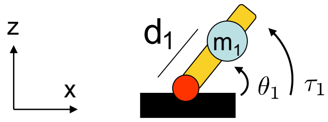


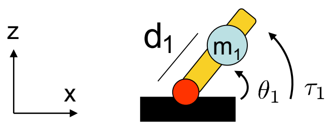
$$x_1 = d_1 \cos\theta_1$$

$$m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos\theta_1 m_1 g + \tau_1$$

Nonlinear!!

What is control input?

Manipulator Control



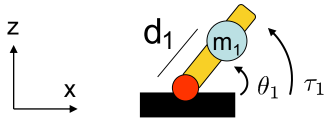
$$x_1 = d_1 \cos\theta_1$$

$$m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos\theta_1 m_1 g + \tau_1$$

Nonlinear!!

How can we apply the linear control method?

Manipulator Control



$$x_1 = d_1 \cos\theta_1$$

$$m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos\theta_1 m_1 g + \tau_1$$

Nonlinear!!

Gravity compensation + Position Control

$$\tau_1 = d_1 \cos\theta_1 m_1 g - K_p(\theta_1 - \theta_{1,r}) - K_d \dot{\theta}_1 \ , \ \dot{\theta}_{1,r} = 0$$