CS 4649/7649
Robot Intelligence: Planning

Roadmap Approaches

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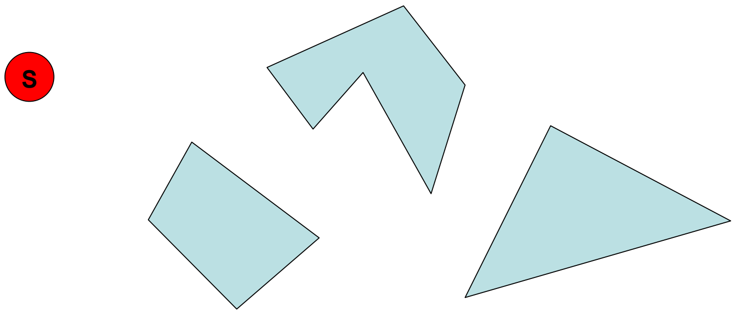
*Slides based in part on Dr. Mike Stilman's lecture slides

Course Info.

- HW#1 due Oct 6th
  - Wiki - Add your group info.
  - Need a repo.?
  - Late policy – No late HWs

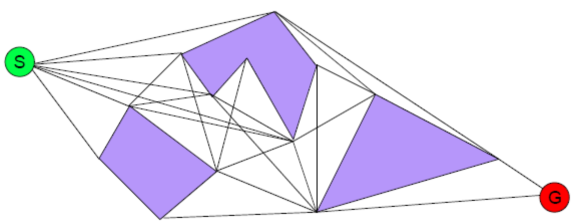
Navigation Planning

Assuming full knowledge, how does the robot 'plan' its path from S to G?



Visibility Graph

- Assuming polygonal obstacles, it looks like the shortest path is a sequence of straight lines joining the vertices of the obstacles

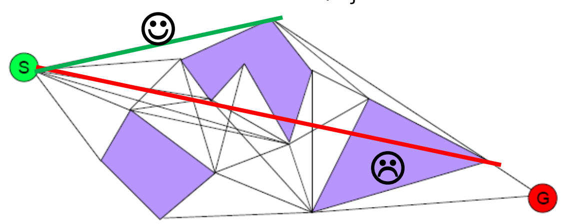


Visibility Graph

- Assuming polygonal obstacles, it looks like the shortest path is a sequence of straight lines joining the vertices of the obstacles
- Visibility graph G = set of unblocked lines between vertices of the obstacles + initial & goal configuration

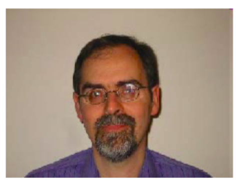
  = (V, E), V ={obstacle vertices} U {start, goal},

  E = {edges (v_i,v_j) disjoint from obstacle interiors}

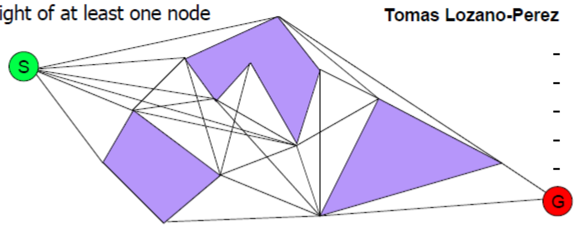


Visibility Graphs*

- Early Motion Planning Algorithm
- Nodes share an edge if they are within "line of sight"
- All points in free space are within sight of at least one node



Tomas Lozano-Perez



- **Point robot**
- **Polygonal obstacles**
- **Build Visibility Graph**
- **Reduce VG**
- **Search**

* "An algorithm for planning collision-free paths among polyhedral obstacles" 1979 T. Lozano-Perez & M. A. Wesley
* "A mobile automaton: An application of artificial intelligence techniques" 1969 N.J Nilson

Path Planning for Robots with Geometric Shapes

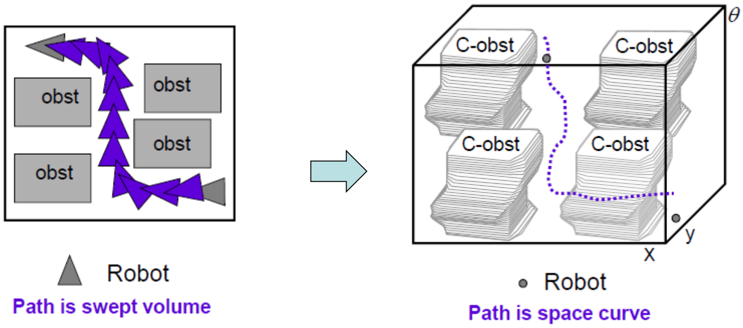
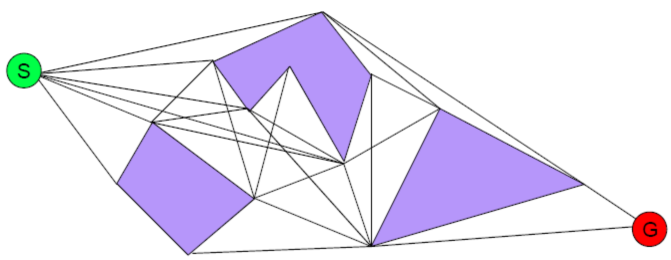


Figure from Dr. Seth Teller's lecture slide

Visibility Graph Analysis

- Visibility Graphs are complete? Yes (Assuming Polygonal Obstacles)
- Visibility Graphs are optimal? Yes Metric: Distance Traveled



Roadmap Approach to Navigation Planning

- Assumption: Static environment
- General Idea:

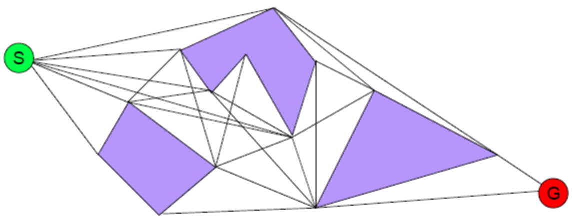
- Avoid searching the entire space
- Pre-compute a (hopefully small) graph (i.e. the roadmap) s.t. staying on the roads is guaranteed to avoid the 'obstacles' (& to take us to the goal)
- Search a path between q_{init} and q_{goal} on the roadmap

Roadmap Approach to Navigation Planning

- Visibility Graph is an example of the roadmap approach
- Assumptions/Limitations? Polygonal objects, Zero distance to obstacles

Do we really care about strict optimality with distance metric?

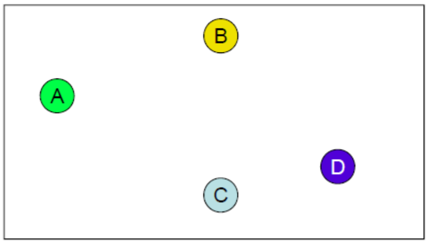
What about safety?



Voronoi Diagrams

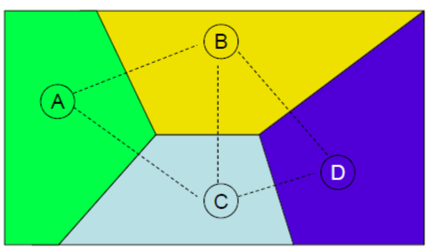
- Another type of 'roadmap'
- Lets color this space
- How would you distribute the continuous space into 4 colors?

Voronoi diagram is a way of dividing space into a number of regions



Voronoi Diagrams

- Space Coloring
  - Associate each point with the area closest to it
  - Boundaries have maximum distance to points



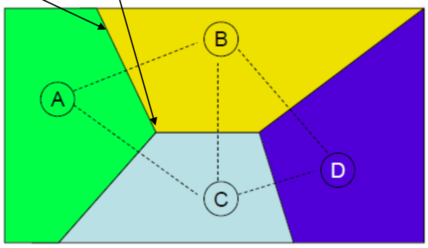
Gregory Voronoi 1868-1908 … student of Andrey Markov (sound familiar?)

- Voronoi Diagram = The set of line segments separating the regions corresponding to different colors

Voronoi Diagrams

Line segment: Points on the edge are equidistant from two data points

Vertices = Equidistant points from >2 data points



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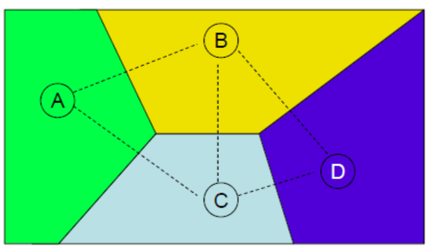
- Voronoi Diagram = The set of line segments separating the regions corresponding to different colors

Voronoi Diagrams

Complexity (in 2D plane) – O(n log n) in time, O(n) in space



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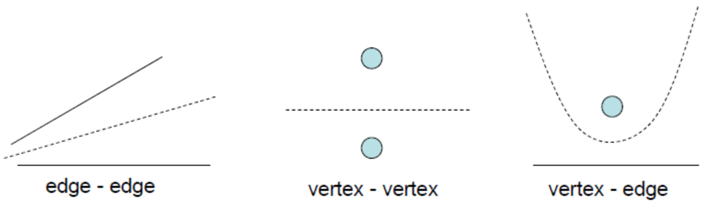
- Voronoi Diagram = The set of line segments separating the regions corresponding to different colors

Generalized Voronoi Diagram (GVD)

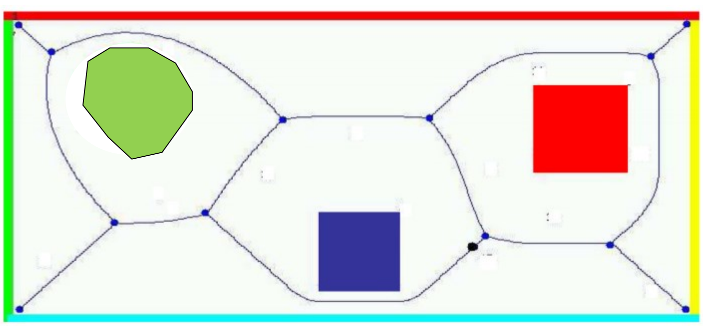
Extending to polygonal objects:

- **Features** of an obstacle are vertices and edges
- A roadmap edge must be between some pair of features



Generalized Voronoi Diagram (GVD)

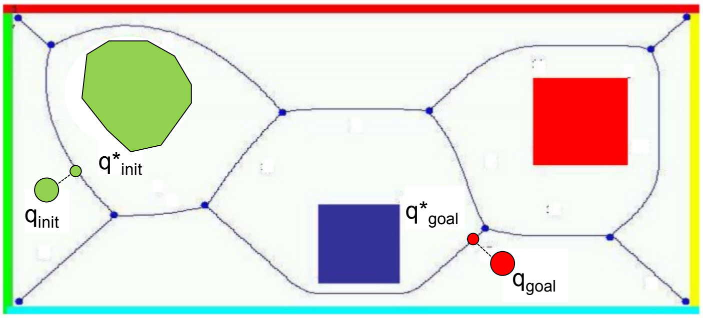
The points on the edges are the furthest from the obstacles & workspace boundary !!



Voronoi Diagram → Navigation Planning

- Idea: Construct a path between q_{init} and q_{goal} by following edges on the Voronoi diagram

Voronoi diagram = Roadmap



Step1. Find the point q^*_{init} of the Voronoi diagram closest to q_{init}

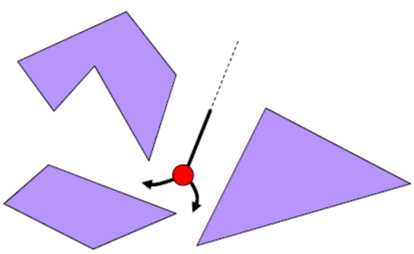
Step2. Find the point q^*_{goal} of the Voronoi diagram closest to q_{goal}

Step3. Compute shortest path from q^*_{init} to q^*_{goal} on the Voronoi diagram

Practical Bug Alternative using GVD

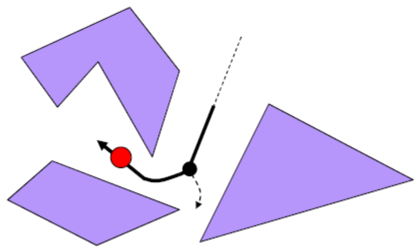
Online Diagram Generation:

- Locate two nearest obstacles and move to midpoint
- Continue along GVG edge until
  - Detect new GVG edges
  - Select one to explore and continue



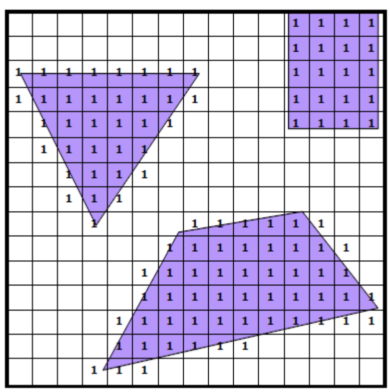
Practical Bug Alternative using GVD

Online Diagram Generation:
- Locate two nearest obstacles and move to midpoint
- Continue along GVG edge until
  - Detect new GVG edges
  - Select one to explore and continue
- Detect boundary points and cycles



Bushfire Algorithm to Generate GVD

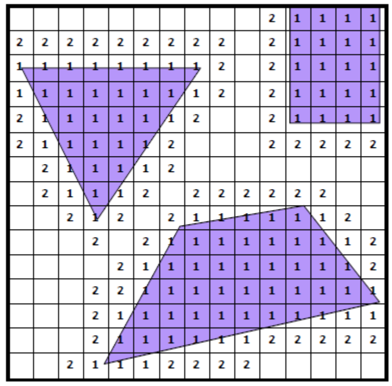
Bushfire GVG Generation
- "Wavefront"
- Start with an empty grid with obstacles = 1



Bushfire Algorithm to Generate GVD

Bushfire GVG Generation

- "Wavefront"
- Start with an empty grid with obstacles = 1
- Expand all cells (i) to (i+1)
- If a cell is expanded twice, label as a GVG edge and do not expand further.



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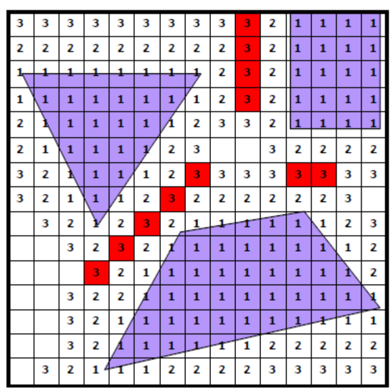

Bushfire Algorithm to Generate GVD

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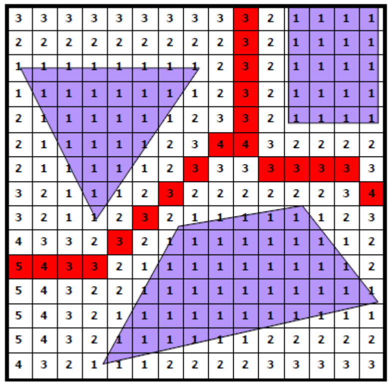
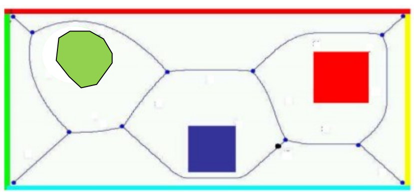


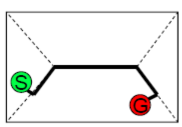
Voronoi Diagrams: Analysis

Completeness:

Yes for Polygonal obstacles

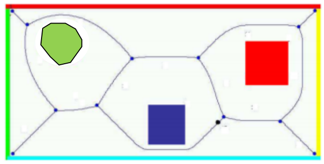


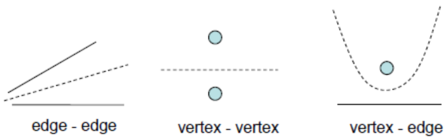
Optimality: ?



Voronoi Diagrams: Summary

- Popular Motion Planning Algorithm
- Edges maximally separate domain features
- Nodes are critical points where edges intersect
- Many practical variants in existance

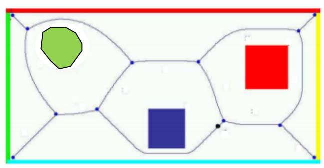


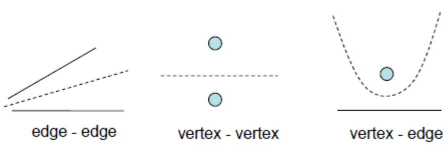


Voronoi Diagrams: Summary

- Difficult in higher dimensions or nonpolygonal worlds
- Use of Voronoi is not necessarily the best heuristic (“stay away from obstacles”)
  - Can be much too conservative
  - Can be unstable:

  Small changes in $C_{obstacle}$ $\rightarrow$ Large changes in GVD





Summary

Roadmap Approach

• Static environment

• Avoid searching the entire space

- Pre-compute a graph (i.e. roadmap) → Search space reduction

Roadmap approach for Navigation Planning

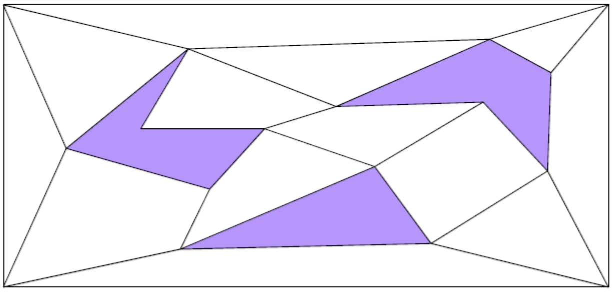
- Visibility Graph → Short path
- Voronoi Diagram → Safe/Conservative path

Other Options

- Exact Cell Decomposition
- Approximate Decomposition
- Potential Fields (Not really grids, but relevant)
- Practical matters: Re-planning

Exact Cell Decomposition: Convex Polygons

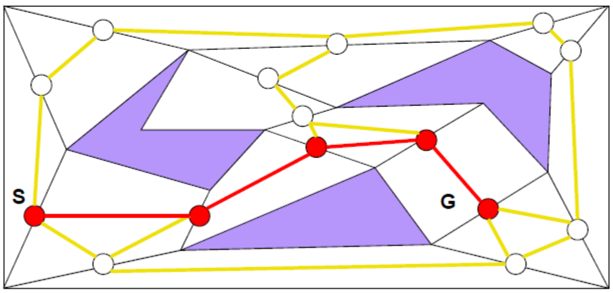
- Collection of non-overlapping cells: Union(Cells) = Free Space
- Finite set of convex polygons that cover the space



Exact Cell Decomposition: Convex Polygons

- Collection of non-overlapping cells: Union(Cells) = Free Space

"The graph of midpoints of edges between adjacent cells defines a roadmap"

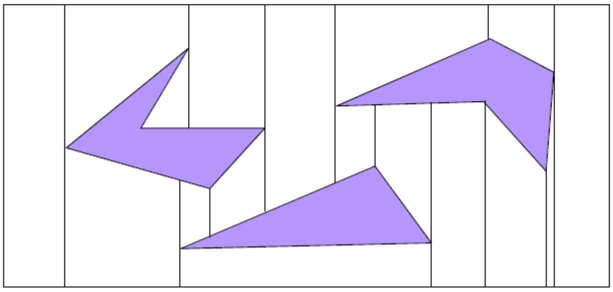


Exact Cell Decomposition: Trapezoidal

- Collection of non-overlapping cells: Union(Cells) = Free Space

Extend a bi-directional vertical line from each vertex until collision

- Gives you trapezoids
- How do we search?

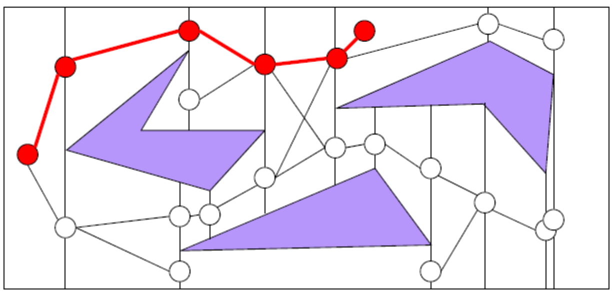


Exact Cell Decomposition: Trapezoidal

- Collection of non-overlapping cells: Union(Cells) = Free Space

Extend a bi-directional vertical line from each vertex until collision

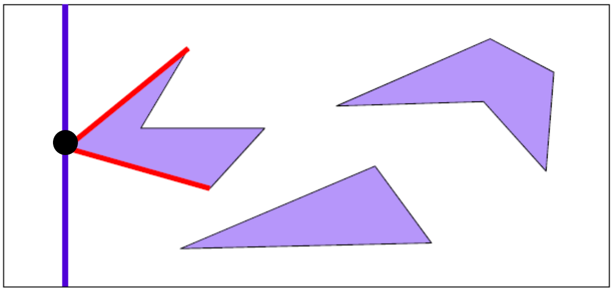
- This **is** a convex polygonal decomposition
- Again a graph search



Exact Cell Decomposition: Trapezoidal

- How do we compute this efficiently?
- Plane Sweep

Critical events: Create new cell, Split cell, Merge cells



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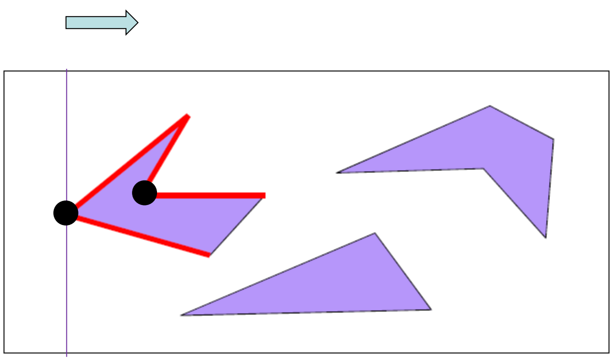

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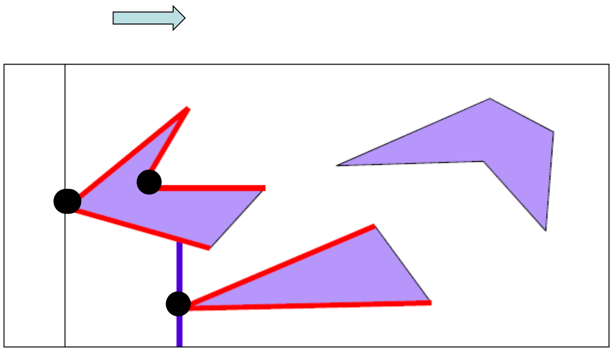

Exact Cell Decomposition: Trapezoidal

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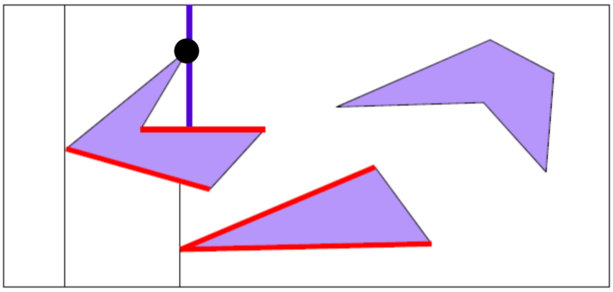

Exact Cell Decomposition: Trapezoidal

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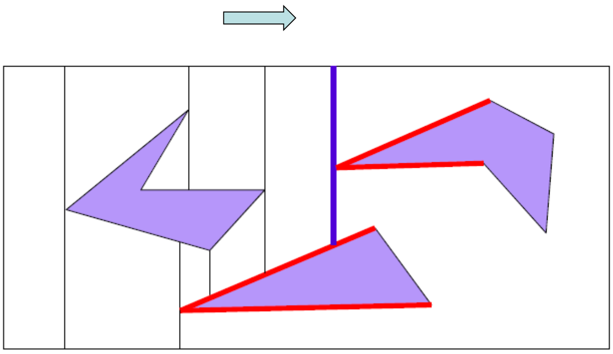
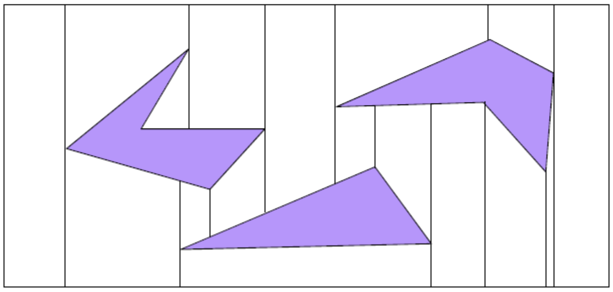


Exact Cell Decomposition: Trapezoidal

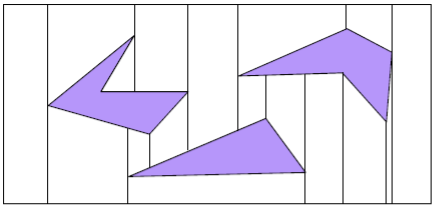
- How do we compute this efficiently?
- Plane Sweep

Complexity (in 2D plane)
- Time: O(n log n)
- Space: O(n)



Exact Cell Decomposition: Analysis

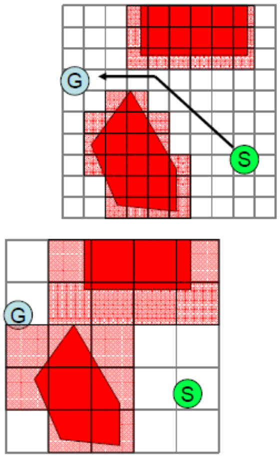
- Complete? Yes
- Optimal? **?**
- Advantage? Efficiency!*



*Expensive and difficult to implement in higher dimensions

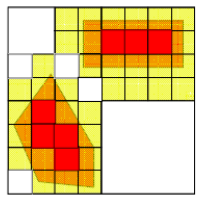
Approximate Cell Decomposition

- Use grid
- Is it complete?
  - Yes, up to grid size
  - **Resolution Complete**
- Is it optimal?
  - Metric = # grid cells traversed
  - Search Method = A*
  - Yes IF **heuristic is admissible**



Better Approximate Cell Decomposition

- 2^m Trees
- m = number of cell subdivisions
- **Only Divide Mixed Cells!**



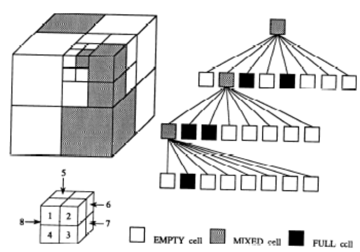
m = 2

$2^2 = 4$

QuadTree

Better Approximate Cell Decomposition

- 2^m Trees
- m = number of cell subdivisions
- **Only Divide Mixed Cells!**

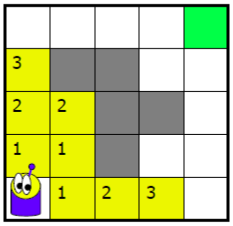


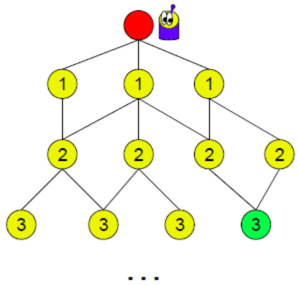
m = 3

$2^3 = 8$

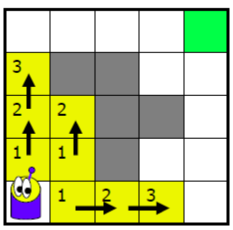
OctTree

Remember BFS: Policy?





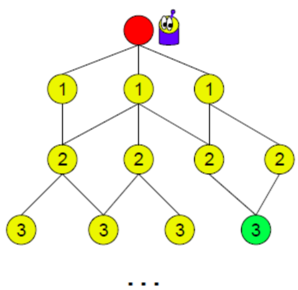
Remember BFS: Policy?



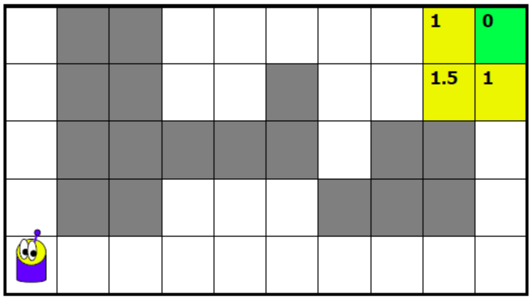
Not a Policy!

BFS / Dijkstra CAN make a policy!

How?



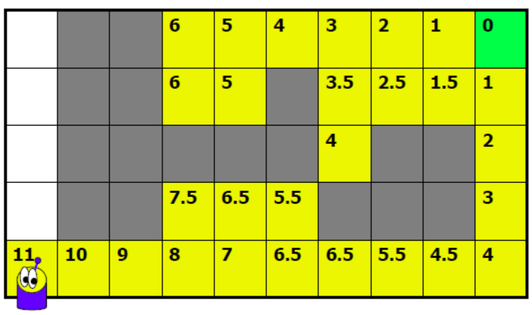
Dijkstra & A*



Dijkstra & A*

A* - Cost = Optimal Cost-to-Come + Heuristic of Cost-to-Go

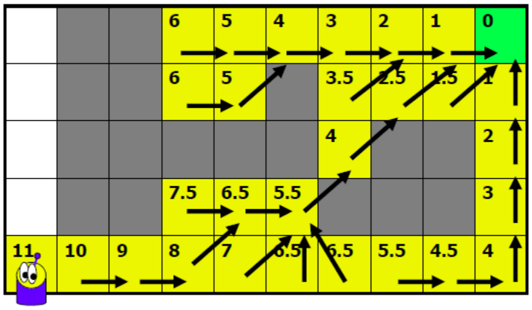
Dijkstra - Cost = Optimal Cost-to-Come



- If a heuristic of 'Cost-to-Go' becomes closer to the true optimal cost-to-go, fewer vertices tend to be explored in comparison with Dijkstra's algorithm
- Dijkstra = A* with Zero-Cost-to-Go (degenerate case of A*)

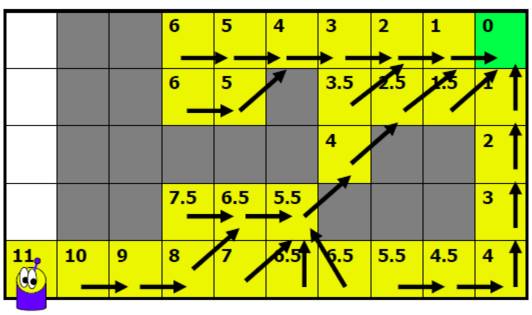
Policy

State → Action



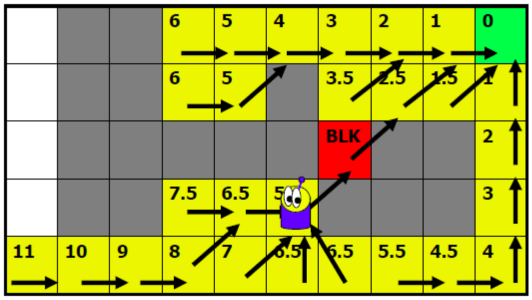
We know what to do at each cell → Policy!

Why is this useful - Replanning



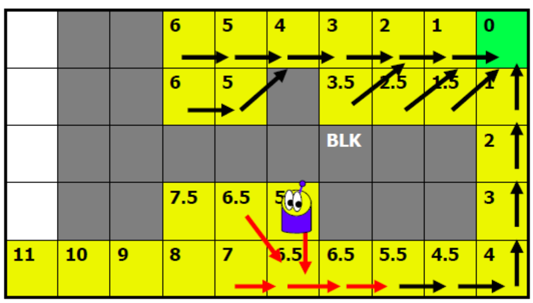
D* / D* Lite

Discovery of an obstacle!



D* / D* Lite Concept

Discovery of an obstacle! Local Updates Re-Create Optimal Plan



- **Maintains cost-to-go values**
- **Propagate backward from the goal → Backward Dijkstra**
- **Useful for navigation in an unknown environment**