The Robot is a Dynamic System
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What is a Dynamic System?
- Mechanical systems, (analog) electrical circuit, ...

How can we describe a dynamic system?

u(t) → x(t) → y(t)

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The Robot is a Dynamic System

What is a Dynamic System?
- A system whose variables are time-dependent
- Linear system: \( \dot{x} = Ax + Bu, y = Cx + Du \)

How can we describe a dynamic system?

Differential equations
Free-body diagrams
Transfer functions
Block diagrams

\[
M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = f(t)
\]

\[
[M \ s^2 + C \ s + K] \ X(s) = F(s)
\]

\[
H(s) = \frac{X(s)}{F(s)} = \frac{1}{M \ s^2 + Cs + K}
\]

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Control

What is Control?

Given the system state, choose input to get a desired output.

If we want to keep \( x = x_d \), what should we do?

Car Speed Control
Cruise Control (Open Loop)

\[ f - m \ddot{x} - b \dot{x} = 0 \]

Steady State \( \rightarrow \) Let \( \ddot{x}_{ss} = 0 \)

\[ 0 + b \ddot{x}_{ss} = f \]

\[ \dot{x}_{ref} = \dot{x}_{ss} = \frac{f}{b} \]

\[ f = b \dot{x}_{ref} \]

State Variable Form:

\[ \dot{x} = \left[ -\frac{b}{m} \right] \dot{x} + \left[ \frac{1}{m} \right] f \]

Advantages?

Disadvantages?

m = 1500kg  \( \quad \) b = 50Ns/m  \( \quad \) \( v_r \) = 27m/s
Car Steering Control

Steering Control (Proportional Control)

State Variable Form:

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & 1/J \\
-K_p & -b/J
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} +
\begin{bmatrix}
0 \\
K_p/J
\end{bmatrix} \theta_r
\]

\[
J = 0.1 \text{kgm}^2 \quad b = 0.1 \text{Nms} \quad \theta_r = 0.5 \text{rad}
\]
Car Steering (Proportional Control)

\[ J\ddot{\theta} + b\dot{\theta} = \tau \]
\[ \dot{\theta} = -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau \]
\[ \tau = -K_p(\theta - \theta_r) \]
\[ \ddot{\theta} = -\frac{b}{J}\dot{\theta} - K_p\frac{1}{J}(\theta - \theta_r) \]

Stability?
(Exponential) Convergence?
Limitations?

Steering Control (PD Control)

\[ J\ddot{\theta} + b\dot{\theta} = \tau \]
\[ \dot{\theta} = -\frac{b}{J}\dot{\theta} + \frac{1}{J}\tau \]
\[ \tau = -K_d(\dot{\theta} - \dot{\theta}_r) - K_p(\theta - \theta_r) \]
\[ \ddot{\theta} = -\frac{b}{J}\dot{\theta} - K_d\frac{1}{J}(\dot{\theta} - \dot{\theta}_r) - K_p\frac{1}{J}(\theta - \theta_r) \]

State Variable Form:

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\theta}_r
\end{bmatrix}
= \begin{bmatrix}
0 & -\frac{K_p}{J} \\
-\frac{b+K_d}{J} & -\frac{b+K_d+K_p}{J}
\end{bmatrix}
\begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\frac{K_p}{J}
\end{bmatrix}\theta_r
\]

J = .1kgm²  b = .1Nms  \theta_r = .5rad
PD Control

Under-damped

Over-damped

Critically damped (Just Right)

PD Control (Selecting Gains)

\[
\ddot{\theta} = -\frac{b + K_d \dot{\theta}}{J} - \frac{K_p (\theta - \theta_r)}{J} \\
\ddot{\theta} = -K_d' \dot{\theta} - K_p' (\theta - \theta_r)
\]

Characteristic Polynomial: \( s^2 + K_d' s + K_p' \)
Standard Form: \( s^2 + 2\zeta w_n s + w_n^2 \)

Damping Ratio: \( \zeta \)
Natural Frequency: \( w_n \)

\[
K_p' = w_n^2 \\
K_d' = 2\zeta w_n
\]

\[
\zeta = 1 \\
K_p' = w_n^2 \\
K_d' = 2w_n = 2\sqrt{K_p'}
\]
Dynamic Response

PD Control (Selecting Gains)

Starting Point: \( K_p' = w_n^2 \)
\[ K_d' = 2w_n = 2\sqrt{K_p'} \]

What is \( w_n \)?

Gains are limited by:
- structural flexibility
- time delay
- sampling rate
- actuator saturation

Optimal Control – Linear Quadratic Regulator

Guess/Nominal values \( \rightarrow \) Tuning!
Cruise Control Revisited (Closed Loop)

\[ f - m\ddot{x} - b\dot{x} = 0 \]

\[ f = -K_p(\dot{x} - \dot{x}_r) \]

F/B control (P control)

Open Loop:

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Fix Steady State Error!

Option 1: Solve for the correct offset (use model)

Option 2: Integral Control (P → I ← D) ... be careful!

Integral Action

PI controller

\[ u(t) = k_p e(t) + k_i \int_0^t e(\tau) \, d\tau \]

Add another ‘corrective’ or ‘restoring’ term, proportional to the integral of error

A constant error e
- yields a constant corrective reaction for proportional action
- yields a growing corrective reaction for integral action

Choice of integral gain
- too small: get asymptotic tracking, disturbance rejection, but very slow
- too large: oscillatory response, or even instability
Integral Action

PI controller

Manipulator Control
Manipulator Control

$x_1 = d_1 \cos \theta_1$

$m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos \theta_1 m_1 g + \tau_1$

Nonlinear!!

What is control input?

Manipulator Control

$x_1 = d_1 \cos \theta_1$

$m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos \theta_1 m_1 g + \tau_1$

Nonlinear!!

How can we apply the linear control method?
Manipulator Control

\[ x_1 = d_1 \cos \theta_1 \]
\[ m_1 d_1^2 \ddot{\theta}_1 = -d_1 \cos \theta_1 m_1 g + \tau_1 \]

Gravity compensation + Position Control

\[ \tau_1 = d_1 \cos \theta_1 m_1 g - K_p (\theta_1 - \theta_{1,r}) - K_d \dot{\theta}_1, \quad \dot{\theta}_{1,r} = 0 \]